1946

PROBABILITY CONCEPTS
IN 11-16 YEAR OLD PUPILS

Report of research
sponsored by the
Social Science Research Council
1978-1981

DR D R GREEN

CAMET

Centre for Advancement of Mathematical Education in Technology
University of Technology
Loughborough, England

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1. AIMS OF THE PROJECT

The aims of this project, which ran from November 1978 to October 1981 were -

a) to survey the intuitions of chance and the concepts of probability possessed by English school pupils aged 11 to 16 years residing in the East Midlands region.

b) to establish patterns of development of probability concepts and to relate these to other mathematics concepts, supplementing the work of the SSRC funded project 'Concepts in Secondary Mathematics and Science' based at Chelsea College, 1974-1979 [Refs. 1,2]

c) to investigate pupil responses to GCE O Level and CSE probability items, with particular reference to sex differences.

2. TEST DEVELOPMENT

A special Probability Concepts Test was devised over a two year period, entailing six pilot versions before achieving the definitive form [Appendix A].

3. DATA SOURCES

a) Altogether about four thousand pupils were involved in the testing although ultimately only 2930 comprised the final 'Sample'. Pupils in East Midlands state comprehensive mixed schools aged 11 to 16 years (i.e. in normal school years 1 to 5) were tested on a class basis with.

i) Probability Concepts Test (about 1 hour) [Appendix A]

ii) NFER AH2 Test of General Reasoning Ability (about 1 hour) [Refs. 3/4]

b) The mathematics teachers of the sample pupils were asked to supply details of each pupil's mathematical ability (M.A.) on a ten-point scale, and also to indicate the texts used and style of teaching adopted.

c) CSE Mathematics Mode 1 scripts were examined, with the permission of the East Midland Regional Examinations Board

1978 N = 1024, 1979: N = 959, 1980 N = 1068

Responses to the probability items were coded and computer analysed. So far the main investigation has concerned sex differences, and much work remains to be done in the future.

d) GCE O Level Syllabus C Mathematics multiple choice responses were supplied in punch card form by the University of London School Examinations Department, as follows

1977 N = 654, 1978. N = 779, 1980 N = 1089

Analysis of the probability items responses was undertaken, with particular reference to sex differences.

4 SAMPLE DETAILS

From the original four thousand subjects, those absent for one test were eliminated and from the remainder a stratified sample was drawn, to reflect the proportions of the various intellectual levels in the English school population as a whole. This was based on the AH2 grade distribution which is purported to be [Refs. 3,4] as follows -

A	B	C	D	E
Top 10%	Next 20%	Middle 40%	Next 20%	Bottom 10%

The final 'Sample' (N = 2930) was as indicated in Tables 1,2,3 and Appendix C

School Year	AH2 Grade					
	A	B	C	D	E	Overall
1	64	128	256	128	64	640
2	67	134	268	134	67	670
3	67	134	268	134	67	670
4	54	108	216	108	54	540
5	41	82	164	82	41	410
TOTAL	293	586	1172	586	293	2930

Table 1 Sample numbers by School Year and by AH2 Grade

School Year	Derbys	Leics	Northants	Notts	Staffs	TOTAL
1	-	468	73	99	-	640
2	86	287	94	178	25	670
3	23	474	45	112	16	670
4	120	283	43	94	-	540
5	23	258	61	68	-	410

Table 2 Sample numbers by School Year and by County

Age Range for School (years)	Number of Schools in Sample	Number of Pupils in Sample
9-13	7	295
10-14	2	146
11-14	12	973
11-16	4	293
11-18	10	729
13-18	2	95
14-18	7	399
	44	2930

Table 3 Sample numbers by School Type

All the participating schools were selected from volounteers, obtained by writing to all relevant schools within 50 miles of Loughborough.

5. PROBABILITY CONCEPTS TESTING PROCEDURE

In order to minimise possible reading difficulties and to ensure that pupils attended properly to each of the 26 questions of the test, and to provide comparable conditions to all classes tested, it was decided that

i) The test would be administered by members of the project team.

ii) All testing would be preceded by a prepared introduction explaining its nature and purpose.

iii) The test would be read out to all 1st, 2nd, and 3rd year pupils (reading time 48 minutes).

iv) The reading out would be so controlled as to give the same amount of time to a question for all pupils

v) The test would be read out to any 4th and 5th year classes for which this was deemed advisable by the class teacher Alternatively, adequate time would be provided (55 minutes minimum) for silent working.

6. PROBABILITY CONCEPTS TEST RESULTS

Pupils' responses to the Test [Appendix A] have been analysed by School Year, and are presented in Appendix B. A summary is included in Section 10.

As a first step towards further analysis of the responses to the Concepts Test, each question was allocated to one of three sub-test categories as follows

Verbal (VB) 7, 13, 14, 15, 16

Combinatoric (CB) 9, 10, 26

Probabilistic (PB) 1, 2, 3, 4, 5, 6, 8, 11, 12, 17 to 25

Each question or part attracted a mark of 1 if correct and 0 if incorrect, yielding three sub-scores (maxima VB 15, CB 6, PB 29) and a total Test Score (maximum 50) For each pupil these four statistics were computed and broken down by School Year, Sex, AH2 Grade, and Mathematical Ability (MA) The test Score breakdowns are shown in Table 4.

	TEST SCORE		VB SCORE		CB SCORE		PB SCORE	
Year	Boy	Girl	Boy	Girl	Boy	Girl	Boy	Girl
1	20 3	19.1	7 2	7.0	2.2	2.2	10.8	9.9
2	22.4	21 2	8.1	7.9	2.4	2.5	11.9	10 7
3	25.5	24 5	8.8	8.9	3.1	3.1	13.6	12.6
4	27.3	25.5	9.7	9.2	3.2	3.3	14.4	13.1
5	28.6	27.4	10 0	9.9	3.6	3.5	16.1	14.1

Table 4 Mean Test Scores broken down by School Year and Sex

An indication of the relative importance to Test Score of age, intelligence and sex is provided by the Analysis of Variance (ANOVA) results presented in Table 5. This shows that all are significant factors. However, the percentage of total variation attributable to each main effect is as follows

Main Effect	Explained Variance
AH2 Grade	44%
School Year	14%
Sex	0.6%

The dominance of intelligence (AH2 Grade) is clear, its influence being three times that of age (School Year) When MA was substituted for AH2 Grade a similar result was obtained (40% variance), indicating that MA is almost as good a guide to Test performance as is AH2 Grade. This is not surprising in view of the high correlation (+ 72) between the two factors.

Source of Variation	F	Significance
Main Effects	485	001
AH2	809	001
Year	271	.001
Sex	107	001
2-Way Interactions	1.74	015
AH2-Year	2.20	.004
AH2-Sex	0.83	.504
Year-Sex	0.96	428
3-Way Interactions		
AH2-Year-Sex	1 02	.433

Table 5 Analysis of Variance for Test Score

7 PROBABILITY CONCEPTS LEVELS

In order to attempt to allocate each pupil unequivocally to a concept 'level', Guttman Scalogram Analysis [Refs. 5,6] was undertaken The aim is to group together selected Concepts Test questions into subtests representing different levels, and to allocate a passmark for each level. To be at a particular level a pupil must pass each lower level and also pass the given level. Guttman Scalogram Analysis is a method whereby the subtests and passmarks can be 'optimised' The main outcome was a scale comprising three levels (plus zero level indicating total failure), as shown in Table 6 Unfortunately, but not surprisingly, many of the more unusual and interesting items in the Concepts Test were eliminated'

Level	Items in Level	Pass Criterion
0	-	Failure at Level 1
1	3, 4, 5, 26a	2/4
2	2, 6b(answer)	7/11
	6b(reason), 6c(answer) 6c(reason), 6d(answer) 9, 10, 18 19(answer), 19(reason)	
3	6d(reason), 6e(answer) 6e(reason)	3/3

Table 6a Details of Probability Concepts Scale

Coefficient of Reproducibility	0.99
Coefficient of Scalability	0.96
Non-Scale cases (Errors/2)	1.3%

Table 6b Statistical details of Guttman Scalogram Analysis for Probability Concepts Scale

The distribution of Levels is shown in Table 7.

		Probability Concept Level				
Year	Sex	0	1	2	3	Mean Level
1	Boy	15	51	31	3	1.21
	Girl	18	60	21	2	1.07
2	Boy	12	44	38	6	1.37
	Girl	11	55	27	6	1 28
3	Boy	5	31	48	16	1.75
	Girl	6	43	40	12	1.57
4	Boy	6	25	44	25	1.87
	Girl	6	34	40	20	1.74
5	Boy	2	16	50	32	2.11
	Girl	3	27	44	26	1 93

Table 7 Percentages of Main Sample Pupils in each Concept Level by School Year and Sex

A steady progression of Concept Level with increasing age is apparent, and confirmed by the means for the five years. Boys show superiority to girls consistently, but the gap between them, as judged by the mean Concept Level, does not widen between the ages of 11-16 years. It would appear that the difference is either innate or, more plausibly, inculcated in the early formative years. Further analysis by intelligence level throws light on the nature of the differences found The relevant breakdowns are presented in Tables 8 and 9 The general trend is that whereas the more able pupils gain about 1 level during the five year period, the least able, starting 1 level behind, fall even further behind during the same period, only gaining about ½ a level. (See also Fig. 1) The average pupil enters secondary school (aged 11) just at Level 1 and leaves (aged 16) at Level 2

	AH2 Grade				
Year	A	B	C	D	E
1	1.84	1 52	1.08	0.80	0.62
2	2.15	1.77	1.30	0.86	0.67
3	2.54	2.12	1 65	1.29	0.67
4	2.67	2.27	1.89	1.25	0.77
5	2.83	2.44	2 10	1.50	1.10
All	2.36	1.98	1.55	1.10	0.74

Table 8 Mean Probability Concept Levels by Year and by General Intelligence (AH2)

	AH2 Intelligence Band		
Year	A/B	C	D/E
1	2	1	0/1
2	2	1	1
3	2/3	1/2	1
4	3	2	1
5	3	2	1

Table 9 Most typical Probability Concept Level for each Year and Intelligence band.

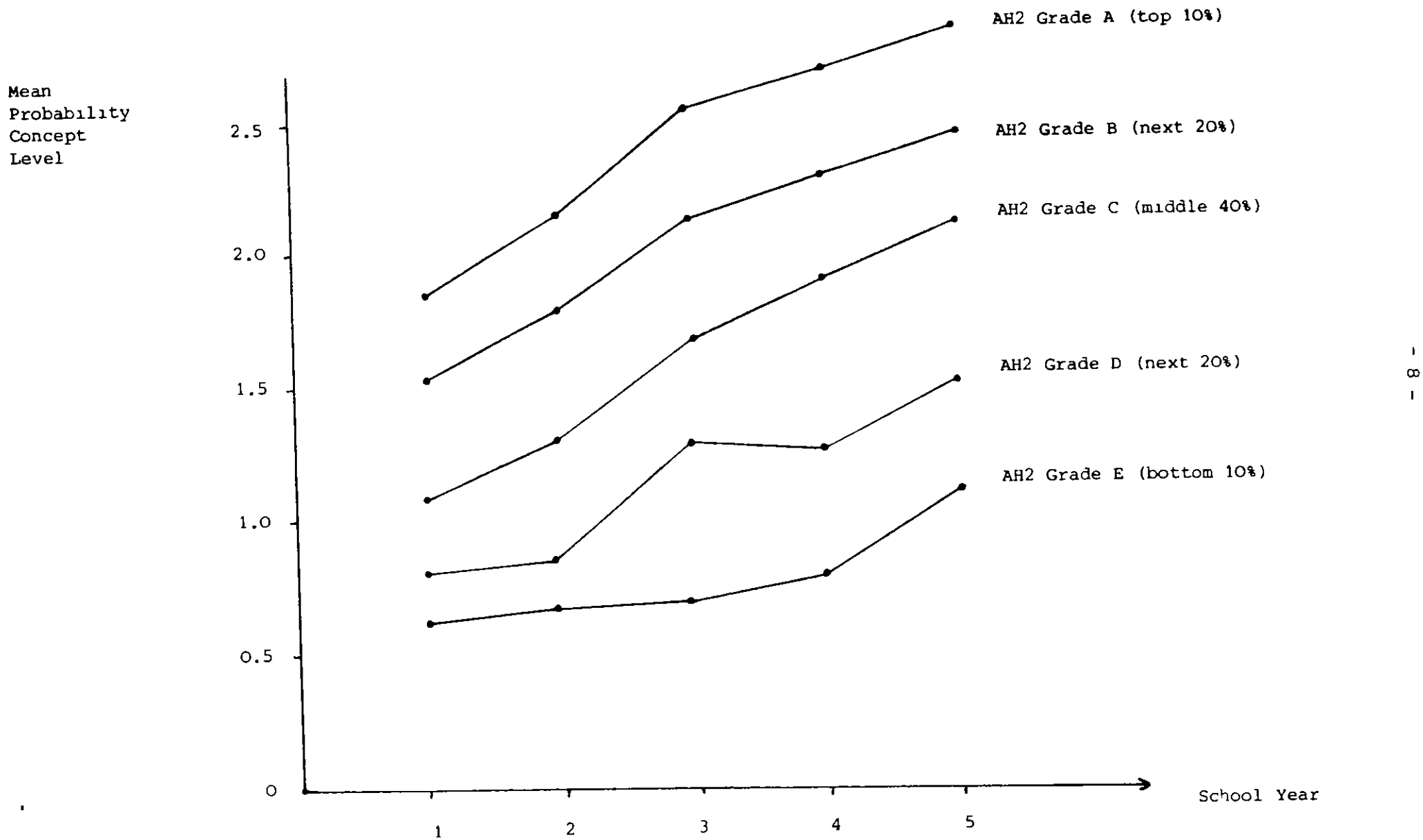


Fig 1 Mean Probability Concept Level profiles by School Year and general reasoning ability.

8. CROSS-REFERENCE TO CSMS

Some pupils in the Sample were subsequently further tested with a CSMS Fractions Test (N = 286) and /or a CSMS Ratio & Proportion Test (N = 169) Thereby an approximate match between CSMS Test Levels and the Probability Concepts Levels could be effected (Table 10). For this purpose a Probability Concepts Level 4 was used, which comprised the combinatoric items 26c,26d, correct answers to both parts being required. (See also Figs 2 to 5 which relate to CSMS Levels in Hart, 1981, Chapter 13)

Probability Level	Associated CSMS Level for given Test: Fractions 1/2	Fractions 3/4	Ratio & Proportion	Corresponding CSMS 'Stage'
0	1	-	1	
1	2	1	1	1
2	3	2	2	2
3	3/4	3	3/4	3/4
4	4	4	4	4

Table 10 Match between Concept Levels in Probability Concepts Test and in CSMS Tests

9. CROSS-REFERENCE TO PIAGETIAN STAGES

By making a detailed study of pupils' responses to the 'comparison of odds items' (Question 6), and relating this to Piaget & Inhelder's own work [Ref. 7] it became possible to indicate the ages at which Piagetian Stages are normally achieved. The results, for three levels of general ability, are shown in Table 11 It must be concluded that most English school children do not achieve the level of formal operations, which is Piaget's Stage III or the Probability Concept Level 3, a result in line with findings on Science concepts reported by Shayer, Kuchemann and Wylam [Ref. 8]

Piagetian Stage	Corresponding Question 6 Item	Sample: Top 30%	Middle 40%	Bottom 30%
IB	(a)	<11	<11	11
IIA	(b)	11	13	16
IIB	(c)	12	14	-
III	(d) (e)	14	-	-

Table 11 Ages at which Piagetian Stages are normally achieved for comparison of odds problems, for three levels of general intelligence

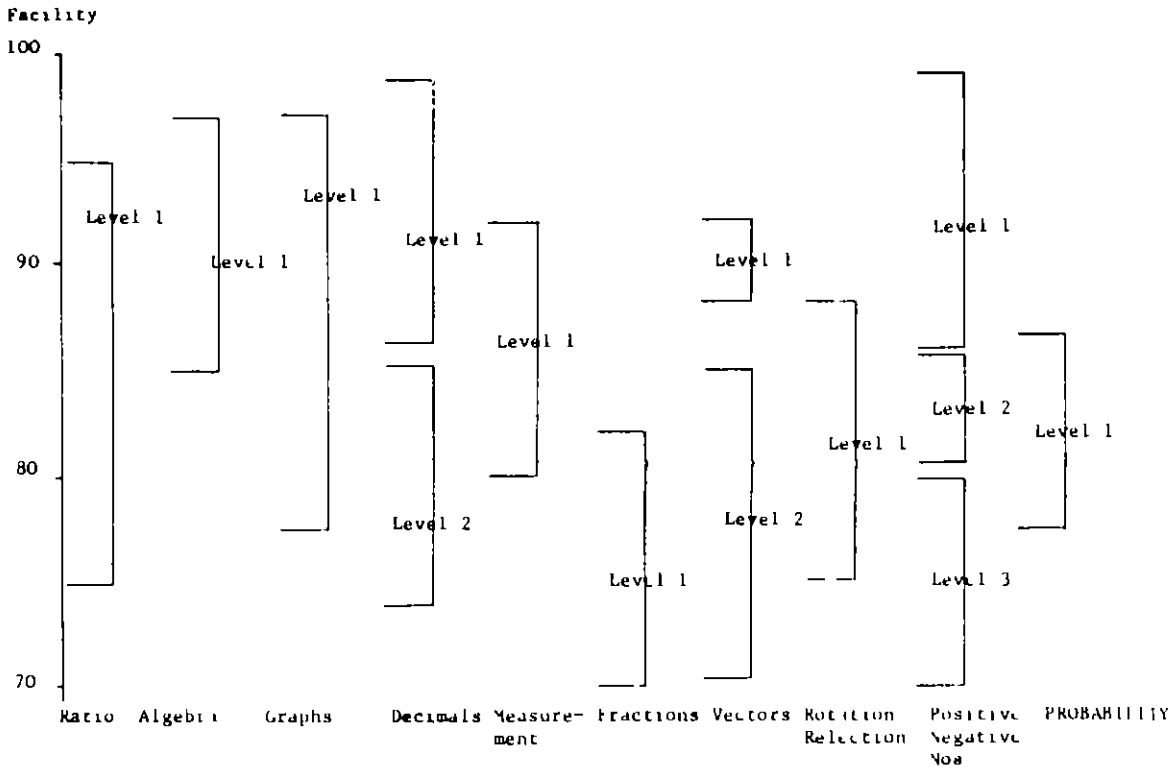
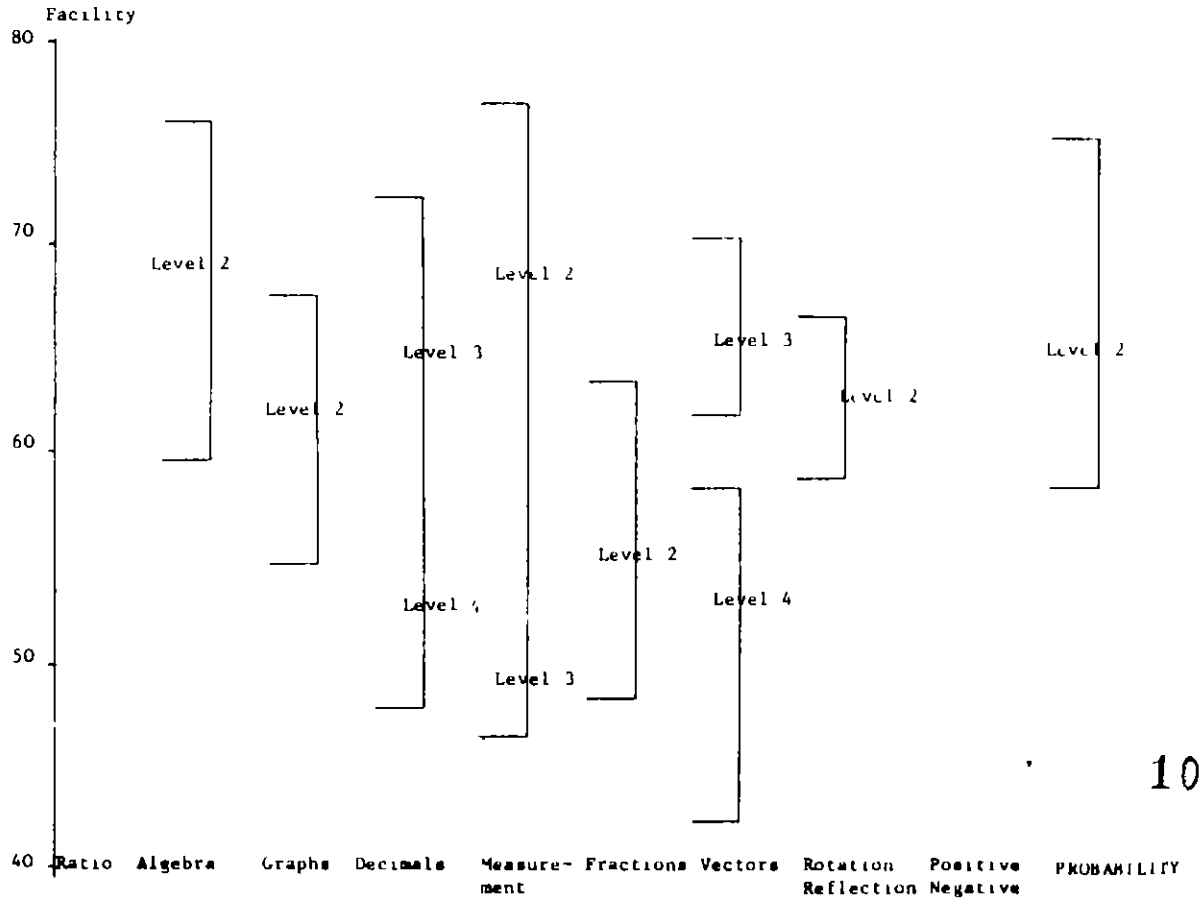


Fig 2 Stage 1 Levels (3rd Year Sample)



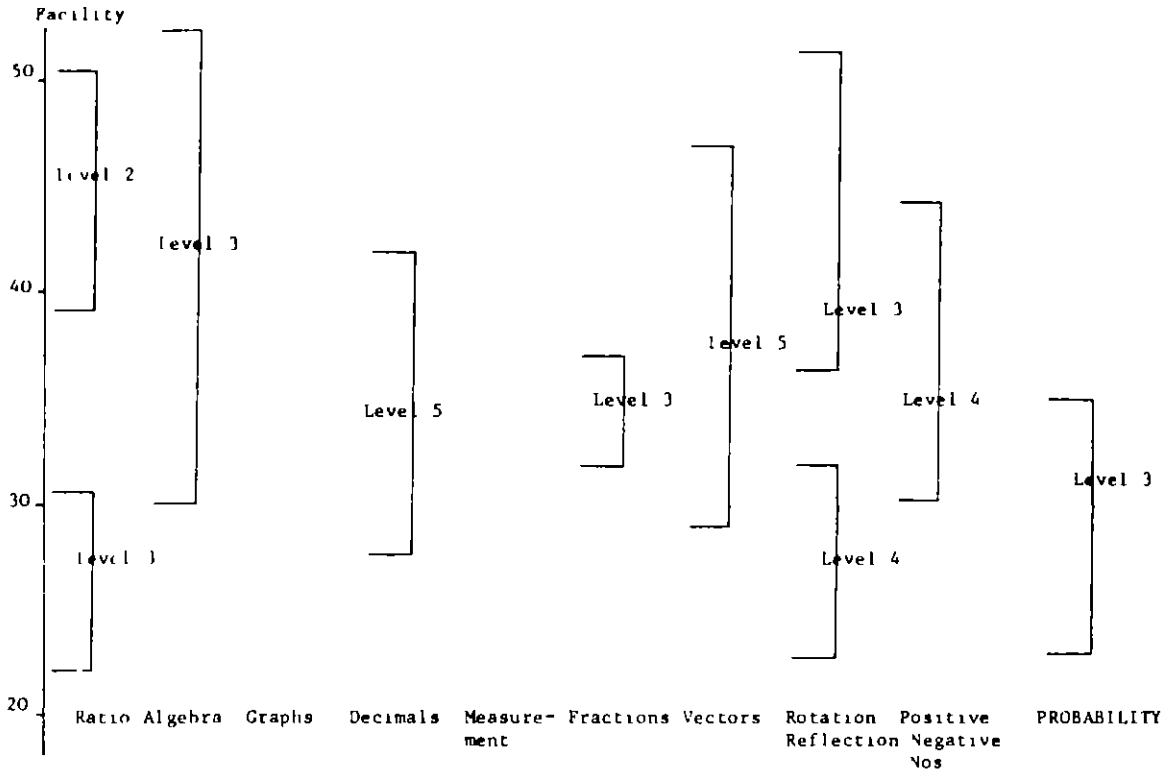


Fig 4 Stage 3 Levels (3rd Year Sample)

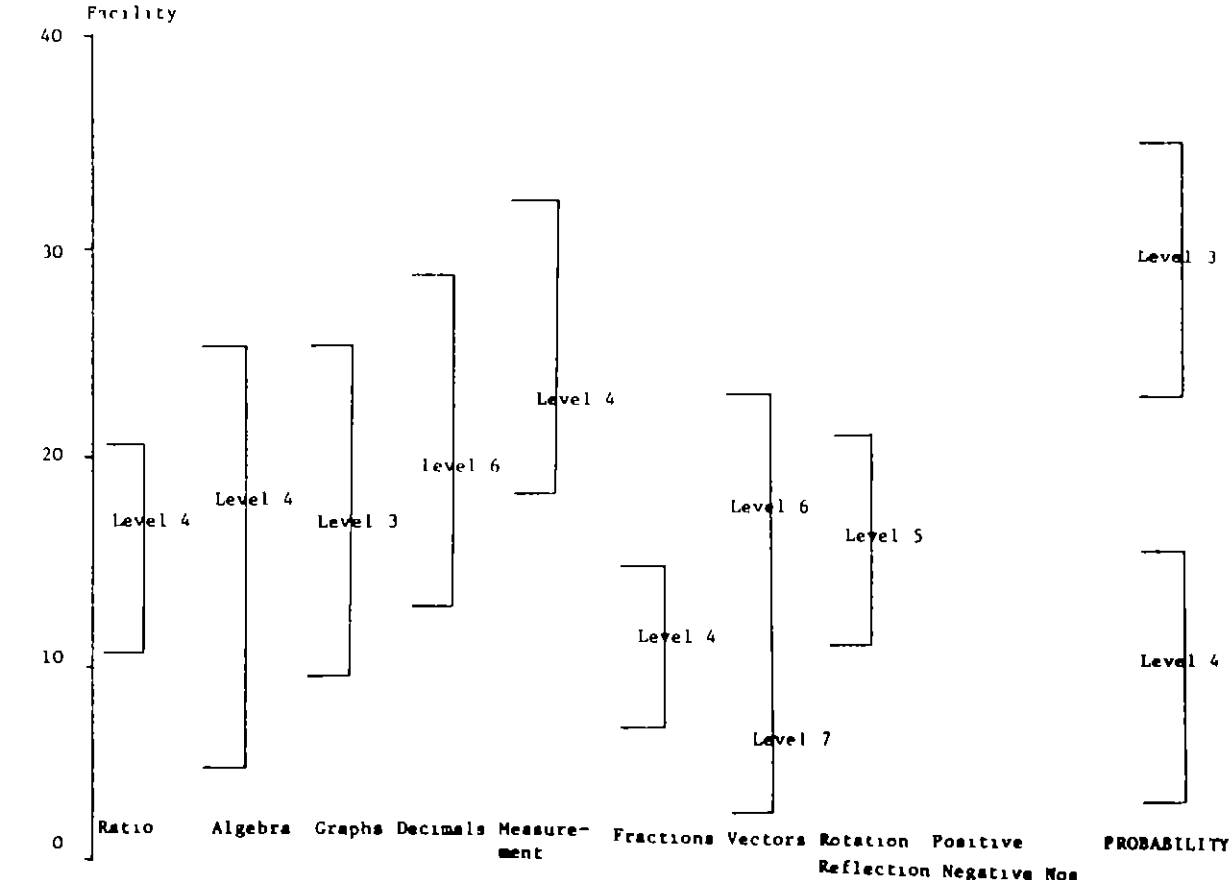


Fig 5 Stage 4 Levels (3rd Year Sample)

10. SUMMARY OF FINDINGS

A SUMMARY OF FINDINGS RELATING TO GCE O LEVEL MATHEMATICS

MULTIPLE CHOICE (LONDON)

This line of research followed up the earlier work of Wood and Brown [Ref. 9] to which the interested reader is referred

1) Probability items are not generally among the harder items
2) Boys perform better than girls on the mathematics paper
3) Boys perform better than girls on probability items
4) Probability items do not have different ranking on facility for boys and girls
5) The percentage difference in facility for probability items between boys and girls is greater than that for the whole paper
6) Boys below the median level perform better on probability items than do girls of comparable mathematical ability
7) Boys above the median level do not perform better on probability items than do girls of comparable mathematical ability.

B SUMMARY OF FINDINGS RELATING TO CSE MATHEMATICS MODE 1 (EMREB)

1) Boys achieve superior grades to girls.
2) Boys achieve superior scores to girls on the Syllabus 1 (options B and C) examination.
3) There is some evidence that boys achieve superior marks to girls on the Syllabus 1 probability questions.
4) Probability items do not exaggerate differences in mathematical ability between the sexes.

C SUMMARY OF FINDINGS RELATING TO THE PROBABILITY CONCEPTS TEST

1) Nearly all test items show an increase in facility with age. (Marked exceptions are Qns. 11, 12, 24).
2) Nearly all test items show an increase in facility with intellectual ability.
3) Often Years 1 and 2 display quite similar response patterns, with substantial increases in performance for Years 3, 4 and 5
4) Items which merely require the counting concept are well done by all Years.
5) Items requiring the ratio concept are very poorly done, particularly below Year 4.
6) Various strategies are employed by pupils in answering comparison of odds problems, as judged by analysis of Question 6 responses. Counting and differencing are often suggested but rarely used consistently Ratio is rarely mentioned but is used by very able, especially older, pupils. (20% of sample got all Qn. 6 correct)
7) Items involving an appreciation of randomness, the stability of frequencies and inference were particularly poorly done, with little evident improvement with age.
8) Although most pupils coped well with a combinatoric item at a concrete level (Qn. 26a) few could proceed to a formal level item (Qn 26c,d).

9) Pupils' verbal ability is often inadequate for accurately describing probabilistic situations.
10) Very often certainty and high probability are equated, as are impossibility and low probability.
11) Pupils freely associate a 50% probability with
    i) outcomes which might or might not happen
    ii) equi-probable outcomes (when more than 2 outcomes exist).
12) Generally, boys consistently outscore girls on total Test Score, but not on the Combinatoric questions (see Table 4)
13) Generally boys show superior performance to girls on many Test questions, with girls significantly superior on one question only (Qn. 20).

14) General reasoning ability (AH2 Grade) is the dominant factor in Test Score (44%), being three times as important as age (School Year) (14%). Sex is of minor significance (½%).

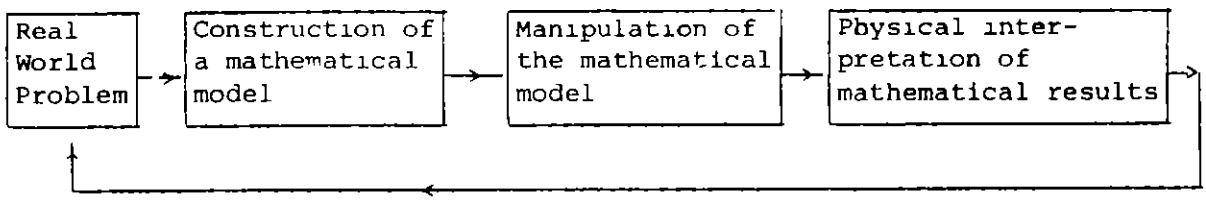
15) The style of teaching, along the 'teacher centred ..... individualised learning' continuum does not appear to be a significant factor affecting Test Score.
16) Generally, boys score more highly than girls on Probability Concepts Level.
17) Probability Concept Level means rise consistently with increasing age (School Year) and intellectual level (AH2 Grade)
18) Most pupils do not attain the level of 'formal operations' (Piagetian Stage III) by Year 5, and presumably leave school at 'concrete operations' level (Stage II).

11. IMPLICATIONS AND RECOMMENDATIONS

In studying the discussion which follows the reader is advised to refer to the Appendices and to Section 10.

a) There is a great need for practical activity from an early age on which to build adequate experience for the pupil. A scheme of guided discovery is called for. This theme is reiterated later in this section.

b) The subject of probability is a very appropriate topic whereby to emphasise the nature of mathematical modelling.


Real World Problem
Construction of a mathematical model
Manipulation of the mathematical model
Physical interpretation of mathematical results

Question 1 is asking the pupil what he thinks is an appropriate model, which dictates his choice of response. (Is colour a relevant factor? Is past history?) Question 5 is similar. A careful consideration of pupils' responses to these two questions is recommended. Are the suggested 'correct' responses in fact correct? Are there other defensible answers?

School mathematics very much emphasises mathematical manipulation and gives very little weight to either model building or interpretation. Is this a proper emphasis?

c) Individually interviewing pupils across the whole age and ability range has confirmed that some pupils are quick to answer questions intuitively, with no obvious intervention of mathematical processes (Qns. 1, 3, 5, 6 particularly) However, other pupils lack this ability and it is clearly unwise for the teacher to assume that all pupils can see what to the teacher may appear obvious, even if some in the class effortlessly provide the 'correct' answers.

Only an extensive programme of class-based activities is likely to provide the weight of evidence and experience to eliminate the fallacious thinking exhibited by pupils and adults alike (Qns. 1,4,5,17,20,24,25 particularly).

d) The more standard aspects of probability clearly require an understanding of fractions and ratio. Ratio in particular, is an essential prerequisite of dealing with the more complex situations [Qns. 2,6].

e) The provision of diagrams seems to help pupils' understanding, and should be provided wherever possible, or pupils encouraged to interpret verbal problems pictorially before proceeding to solve them

f) Various strategies are employed by pupils in solving comparison of odds questions (Qn 6) but little consistency can be found. Much benefit can accrue from explicit classroom discussion of appropriate strategies for particular problems with a view to developing adequate discernment of types of problem or of mastery of a universally applicable strategy

g) The verbal weakness of pupils has been exposed by questions 7, 13 to 15.

The teacher needs to give attention to educating pupils in the normally accepted usage of various terms which indicate varying degrees of likelihood Commonality of meaning is obviously most desirable, and should not tacitly be assumed. Discussion of events incorporating a chance element needs to be encouraged.

h) The idea of expectation (Qn. 9) seems to be fairly widely available despite its rarely being met in lessons. It seems natural to capitalise on this important concept in introducing probability as a meaningful study of the real world.

i) In our scientifically and technologically dominated society, with its deterministically oriented education system, it is perhaps inevitable that the concept of randomness should be neglected. Pupils' responses to Qns 11,12,20, for example, reflect this Very little time or attention is given to this concept despite the fact that we are surrounded by random occurrences every day of our lives. Is it reasonable to treat so lightly such a pervasive property of our world?

j) It might be thought that with the emphasis given to symmetry in school mathematics pupils would have little difficulty in handling problems where symmetry was a key factor. That this is not nessarily so is exhibited by responses to Qns 6c,16,17,21. This may reflect the gap between a theoretical study and its application.

k) Although Fischbein (Ref. 11) has shown that the multiplication law can be well understood by a careful teaching programme utilising tree diagrams, our evidence is more in accord with most research which indicates that the multiplicative principle is poorly understood, pupils having little in the way of basic intuition on which it can be based. Response to Qn 22 was very poor. Even pupils who had recently studied tree diagrams said, in interview, that tree diagram work did not help them (i.e. was not relevant).

l) Drawing inferences from sampling experiments was another very weak area, as judged by responses to Qns 23,25. Pupils tended to rely on the abstract idea of 'equality' and not relate it to the physical situation. A lack of experience in this area may be a factor - clearly pupils need to be given such experience, and the interest shown in the media for opinion polls is clearly am excellent basis on which to build.

m) The stability of frequencies (i e. the evening out of proportions with increasing numbers of trials) is a most important concept. Those questions relating to this topic (Qns 8,24) were not wholly satisfactory because of inevitable language problems. The results, and follow-up interviews, suggest that a lot of practial activity and discussion is needed to develop an appreciation of this concept in the majority of pupils.

12. FURTHER RESEARCH

Much further research in this area is called for. Anyone interested in pursuing this, on however small a scale, is invited to write to David Green or to Professor A C Bajpai at CAMET.

13. FURTHER DETAILS

Greater detail of this research may be obtained by consulting Ref 12 or from the author.

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CHANCE AND PROBABILITY CONCEPTS PROJECT

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SCHOOL ....

TODAY'S DATE . ...

BOY/GIRL .. . .

AGE . . .. . . years

DATE OF BIRTH . . ..

MATHS SET . . ..

YEAR .

NAME . . . .

1. A small round counter is red on one side and green on the other side It is held with the red face up and tossed high in the air. It spins and then lands. Which side is more likely to be face up, or is there no difference? Tick the correct answer

(A) The red side is more likely ☐

(B) The green side is more likely ☐

(C) There is no difference ☐

(D) Don't know ☐

2. A mathematics class has 13 boys and 16 girls in it Each pupil's name is written on a slip of paper All the slips are put in a hat The teacher picks out one slip without looking Tick the correct sentence

(A) The name is more likely to be a boy than a girl ☐

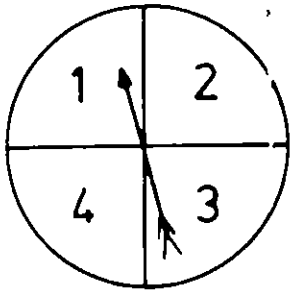
(B) The name is more likely to be a girl than a boy ☐

(C) It is just as likely to be a girl as a boy ☐

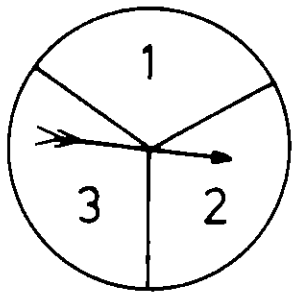
(D) Don't know ☐

3. Here are pictures of two discs which have pointers which are spun and point to a number. With which disc is it easier to get a 3?

Tick the correct answer



RED



BLUE

(A) It is easier to get 3 on the Red disc ☐

(B) It is easier to get 3 on the Blue disc ☐

(C) The two discs give the same chance of getting a 3 ☐

(D) Don't know ☐

Why did you chose this answer? . . .

. .

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4 When an ordinary 6 sided dice is thrown which number or numbers is it hardest to throw, or are they all the same?

ANSWER . . . .

---

5 An ordinary coin is tossed <u>five times</u> and 'Heads' appears <u>every time</u>

Tick the correct sentence below

(A) Next time the coin is more likely to turn up 'Heads' again ☐

(B) Next time the coin is more likely to turn up 'Tails' ☐

(C) Next time 'Heads' is as likely as 'Tails' ☐

(D) Don't know ☐

6(a) Bag A has got in it 3 black counters and 1 white counter.
Bag B has got in it 2 black counters and 1 white counter
(See the diagram)

If you have to pick a <u>black</u> counter to win a prize and you must not look in the bag, which bag should you choose to pick from?

Tick the correct answer.

A

B

(A) Bag A gives a better chance to get black ☐

(B) Bag B gives a better chance to get black ☐

(C) Both bags give the same chance ☐

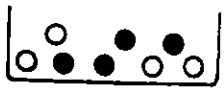
(D) Don't know ☐

Why? .. ..

6(c) Two other bags also have black and white counters

Bag E 2 black and 2 white

Bag F 4 black and 4 white



Which bag gives a better chance of picking a black counter?

(A) Bag E ☐
(B) Bag F ☐
(C) Same chance ☐
(D) Don't know ☐

Why? .. ..

6(d) Two other bags have black and white counters

Bag G 12 black and 4 white
Bag H 20 black and 10 white

Which bag gives a better chance of picking a black counter?

(A) Same chance ☐
(B) Bag G ☐
(C) Bag H ☐
(D) Don't know ☐

Why? . . ..

6(e) Two other bags have black and white counters

Bag J 3 black and 1 white
Bag K 6 black and 2 white

Which bag gives a better chance of picking a black counter?

(A) Same chance ☐
(B) Bag J ☐
(C) Bag K ☐
(D) Don't know ☐

Why? ....

7(a) Here are five phrases

1	Cannot happen
2	Does not happen very often
3	Happens quite often
4	Happens nearly every time
5	Always happens

For this question you must put one number in each of the four boxes below You can use the same number more than once if you wish.

(i) Which of these phrases means the same as "Very likely"? ☐

(ii) Which of these phrases means the same as "Unlikely"? ☐

(iii) Which of these phrases means the same as "Likely"? ☐

(iv) Which of these phrases means the same as "Not very likely"? ☐

7(b) Give a word or phrase which means the same as the following

(i) Impossible

(ii) Possible

(iii) Even chance

(iv) Little chance

(v) Very probable

8 In an experiment 12 coins are all tossed up in the air together, and land on a table. If the experiment is repeated a lot of times which one of the following results will happen most often?

(A) 2 heads and 10 tails ☐

(B) 5 heads and 7 tails ☐

(C) 6 heads and 6 tails ☐

(D) 7 heads and 5 tails ☐

(E) All have the same chance ☐

---

9 Mark and Steven play a dice game.

Mark wins 1 penny if the dice comes up 2 or 3 or 4 or 5 or 6.
If it comes up 1 Steven wins some money How much should Steven win when he throws a 1 if the game is to be fair?

ANSWER . . .. p

---

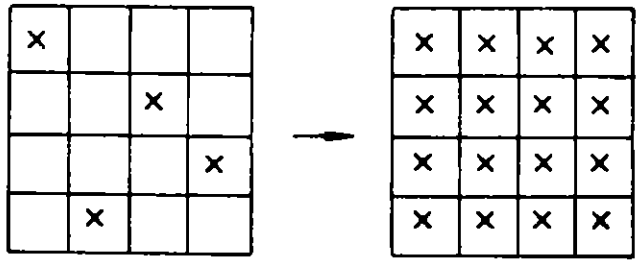
10 A 2p coin and a 10p coin are tossed together One possible result, Heads on the 2p coin and Tails on the 10p coin, has already been put in the table. H for heads and T for tails. Write in all the other possible results.

2p coin	10p coin
H	T

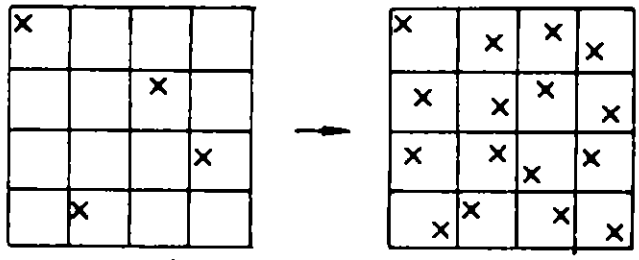
---

11 The flat roof of a garden shed has 16 square sections It begins to snow At first just a few snowflakes fall, then after a while more have landed Below are three sets of two pictures Each set shows the pattern of snowflakes building up - first 4 flakes, then 16 flakes

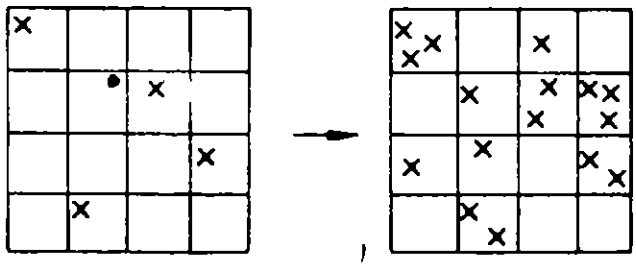
SET A



SET B



SET C



QUESTION Which of these sets best shows the kind of pattern you would expect to see as the snowflakes land?

Set A ☐

Set B ☐

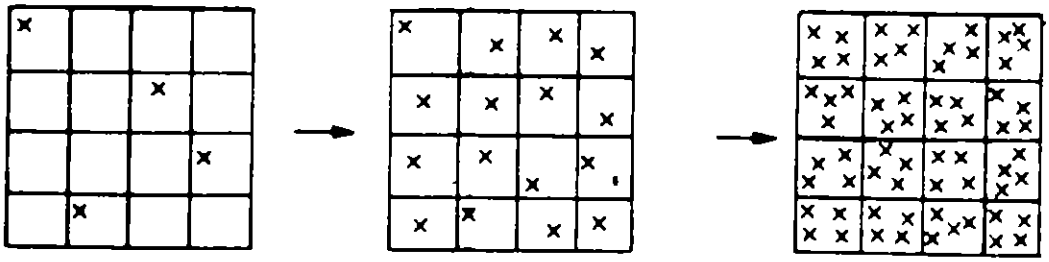
Set C ☐

Sets B and C ☐

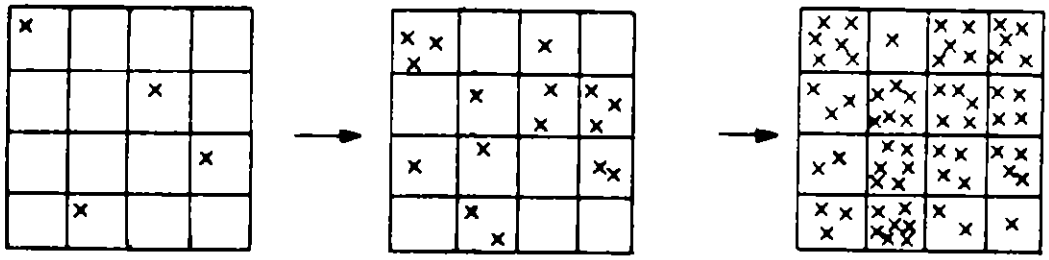
Each kind of pattern is as likely ☐

12 **Below are three sets of three pictures. Each set shows the pattern of snowflakes building up - first 4 flakes, then 16 flakes, then 64 flakes.**

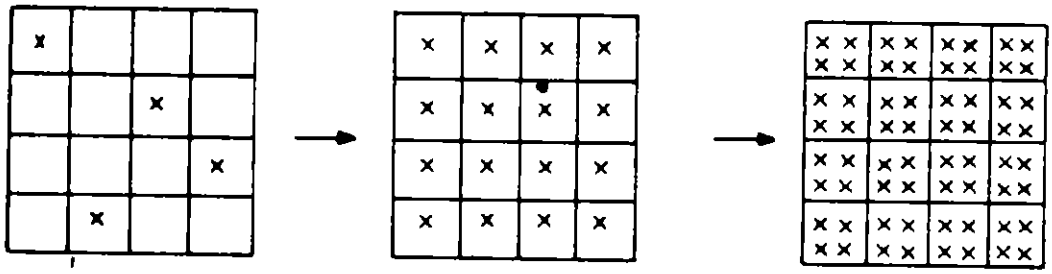
SET A



SET B



SET C



QUESTION Which of these sets best shows the kind of pattern you would expect to see as the snowflakes build up?

Set A ☐

Set B ☐

Set C ☐

Sets A and B ☐

Each kind of pattern is as likely ☐

13 Write a sentence which begins "It is very likely that the Queen ....." using your own words to finish it.

It is very likely that the Queen ....

. . . .

---

14 Write a sentence which begins "It is unlikely that the Queen ....." using your own words to finish it.

It is unlikely that the Queen ....

. ...

---

15 Write a sentence which ends "is something that happens by chance" using your own words to start it

. . ..

. . .. is something that happens by chance.

---

16 For the following phrases tick all those which you think mean exactly the same as "has a 50-50 chance of happening"

(A) It may happen or it may not ☐

(B) It has an even chance of happening ☐

(C) It will happen 50 times out of 50 ☐

(D) It can happen sometimes ☐

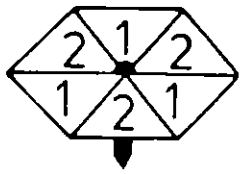
(E) It has an equal chance of happening or not happening ☐

(F) It is very unlikely to happen ☐

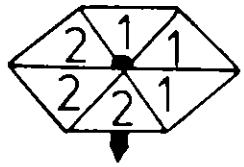
---

17 Two six-sided spinners are marked with '1's and '2's, as in the diagrams

Yellow



Red



Which spinner gives you a better chance of landing on a '2' when it spins or do they give the same chance?

(A) Yellow is better for getting a 2 ☐

(B) Red is better for getting a 2 ☐

(C) Both spinners give the same chance ☐

(D) Don't know ☐

Why?

---

18 4 red marbles, 4 blue marbles and 2 green marbles are put into a bag which is then shaken. Three marbles are picked out - 2 red and 1 blue. Then one more marble is picked out. Which colour is it most likely to be?

(A) Red has the best chance ☐

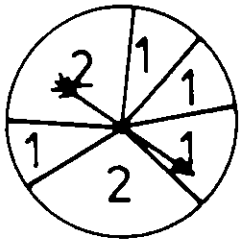
(B) Blue has the best chance ☐

(C) Green has the best chance ☐

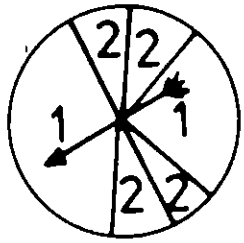
(D) All colours have the same chance ☐

(E) Don't know ☐

19(a) Two discs, one orange and one brown, are marked with numbers.



Brown



Orange

Each disc has a pointer which spins round. If you want to get a 1, is one of the discs better than the other, or do they both give the same chance?

(A) Brown is better for getting a 1 ☐

(B) Orange is better for getting a 1 ☐

(C) Both discs give the same chance ☐

(D) No one can say ☐

19(b) Why did you choose this answer?

. .. .

---

20 A teacher asked Clare and Susan each to toss a coin a large number of times and to record every time whether the coin landed Heads or Tails. For each 'Heads' a 1 is recorded and for each 'Tails' an 0 is recorded.

Here are the two sets of results

CLARE
0101100110010101101101000111000110110101010110010001
0101001110011010110010110010110010010111011001 1011
0101001011001010110001001101011001110111010110001 1

SUSAN
1001110111101001110010011100100011101111110101010 1
1110000001000101001000001000110001010000000011001
0000000111110000110101001001001111110100110001 1000

Now one girl did it properly, by tossing the coin. The other girl cheated and just made it up.

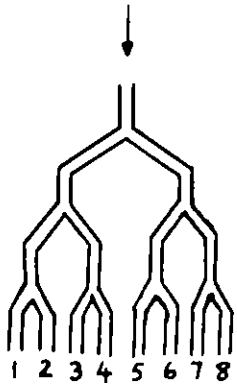
(a) Which girl cheated? ANSWER .....

(b) How can you tell?

ANSWER ....

. . . .

21 (a) Suppose a lot of marbles are dropped down the set of channels drawn below



Tick the sentence which best describes where you expect the marbles to go.

(A) Each channel will get about the same number of marbles ☐

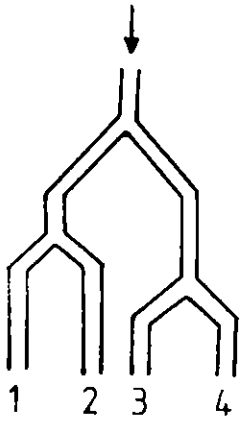
(B) 1 and 8 will get the most marbles ☐

(C) 3, 4, 5 and 6 will get the most marbles ☐

(D) 1, 3, 5 and 7 will get the most marbles ☐

(E) None of these ☐

(b) Now do the same for this set of channels



(A) Each channel will get about the same number of marbles ☐

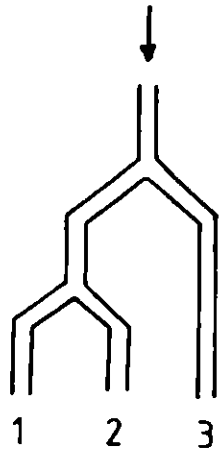
(B) 1 and 2 will get the most ☐

(C) 3 and 4 will get the most ☐

(D) 1 and 4 will get the most ☐

(E) None of these ☐

(c) Now do the same for this set of channels



(A) Each channel will get about the same number of marbles ☐

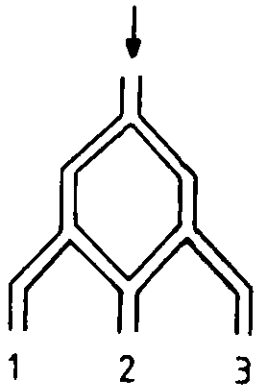
(B) 2 will get the most marbles ☐

(C) 3 will get the most marbles ☐

(D) 1 and 2 will get a lot of marbles and 3 will get a few ☐

(E) None of these ☐

(d) Now do the same for this set of channels



(A) Each channel will get about the same number of marbles ☐

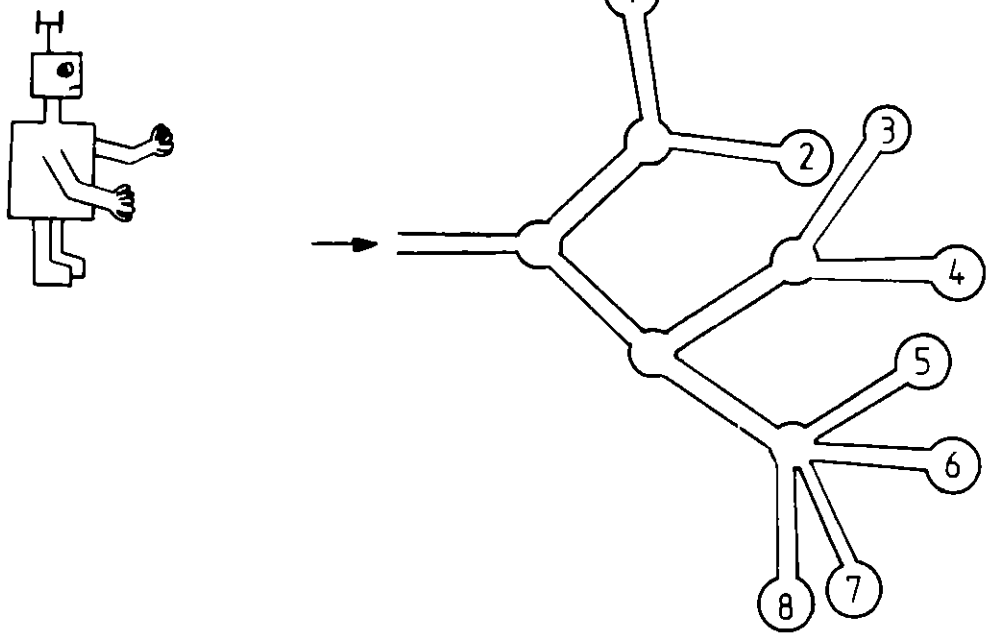
(B) 2 will get about twice as many marbles as 1 or 3 ☐

(C) About half will go down 1 and about half down 3 ☐

(D) A few will go down 1, nearly all down 2 and a few down 3 ☐

(E) None of these ☐

22 A robot is put into a maze, which it begins to explore. At each junction the robot is as likely to go down any one path as any other (except it does not go back the way it came). There are eight traps at the ends of the eight paths (see picture). In which trap or traps is the robot most likely to finish up, or are all traps equally likely?



ANSWER . .

---

23 A packet of 100 drawing pins was emptied out onto a table by a teacher. Some drawing pins landed 'UP' and some landed 'DOWN'.

The result was UP 68, DOWN 32

Then the teacher asks a girl to repeat the experiment.

Choose from the list below the result you think the girl will get.

(A) UP 36 DOWN 64 ☐

(B) UP 63 DOWN 37 ☐

(C) UP 51 DOWN 49 ☐

(D) UP 84 DOWN 16 ☐

(E) All these results have the same chance ☐

---

24 Which of the following results is more likely?

(1) Getting 7 or more boys out of the first 10 babies born in a new hospital

(2) Getting 70 or more boys out of the first 100 babies born in a new hospital

(A) They are equally likely ☐

(B) 7 or more out of 10 is more likely ☐

(C) 70 or more out of 100 is more likely ☐

(D) No one can say ☐

---

25 A bag has in it some white balls and some black balls A boy picks out a ball, notes its colour, and puts it back. He then shakes the bag He does this four times. He picks a black ball every time.

He then picks out one more ball. What colour do you think he is likely to get?

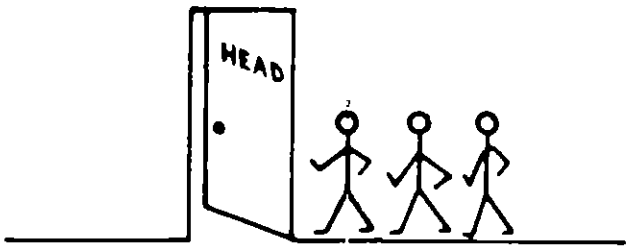
Tick the correct sentence

(A) Black is more likely again ☐

(B) Black and white are equally likely ☐

(C) White is more likely this time ☐

26



Three boys are sent to the headmaster for stealing. They have to line up in a row outside the head's room and wait for their punishment No one wants to be first of course'

(a) Suppose the boys are called Andy, Barry and Christopher (A, B, C for short) We want to write down all the possible orders in which they could line up For example A B C we write ABC as shown below

1st 2nd 3rd

ANSWER ..ABC.. / .. .. /... .. ./ . . ./ .. ./ .. ./ . .. ./

..... / .... . / . .. /. . .. ./ ... / . .. ./ . ../ . . /

Now write down all the other different orders.

(b) How many different ways are there altogether? ANSWER . . . . . .

Now do the rest of the question.

(c) If four boys (A, B, C, D) have to be lined up, how many different ways are there? ANSWER ... . ..

(d) And if there are five boys how many ways?
(Don't try to write them all down!) ANSWER .. . .

TEST RESULTS - ITEM RESPONSES

* indicates the response(s) considered to be correct

QN1

Year	A	B	C*	D
1	18	28	45	9
2	15	24	55	6
3	11	20	66	2
4	12	13	71	4
5	11	12	74	4
All	14	20	61	5

QN2

Year	A	B*	C	D
1	5	38	53	4
2	4	43	51	2
3	4	61	35	0
4	2	60	37	1
5	4	71	25	0
All	4	53	42	2

QN3(a)

Year	A	B*	C	D
1	4	71	22	3
2	4	76	16	3
3	2	87	11	1
4	3	87	9	1
5	3	92	5	0
All	3	82	13	2

QN3(b)

Year	Area Concept	Counting Concept	Ratio Concept	Position or speed	Other	Omit
1	28	28	2	13	19	10
2	33	34	2	9	16	8
3	39	33	6	7	9	7
4	39	30	7	3	7	12
5	39	28	9	5	6	13
All	35	31	4	8	13	10

QN4

Year	1	2	3	4	5	6	*'Same'	6+ Other	Other	Omit
1	1	0	0	0	0	23	67	2	1	5
2	2	0	0	0	0	22	69	2	1	3
3	1	0	0	0	0	13	79	4	1	2
4	2	0	0	0	0	14	79	3	1	0
5	0	0	0	0	0	9	86	2	1	1
All	1	0	0	0	0	17	75	3	1	2

QN5

Year	A	B	C*	D
1	14	14	67	5
2	10	14	73	3
3	11	10	78	1
4	10	10	78	1
5	10	9	80	1
All	11	12	75	2

QN6(a)

Year	A*	B	C	D
1	88	4	6	2
2	87	6	6	0
3	88	5	7	0
4	90	4	5	0
5	90	5	5	0
All	88	5	6	1

QN6(b)

Year	A*	B	C	D
1	55	9	33	2
2	60	9	31	1
3	71	5	23	1
4	75	4	21	0
5	81	3	15	1
All	67	6	25	1

Qn 6(c)

Year	A	B	C*	D
1	11	44	43	3
2	12	36	51	1
3	9	26	64	1
4	8	18	73	1
5	5	14	80	0
All	9	29	60	1

Qn 6(d)

Year	A	B*	C	D
1	8	38	51	2
2	6	50	41	4
3	6	62	31	1
4	5	68	26	0
5	4	74	20	2
All	6	57	35	2

Qn 6(e)

Year	A*	B	C	D
1	20	14	61	2
2	21	16	58	3
3	35	16	46	2
4	42	13	44	1
5	56	9	33	1
All	33	14	50	2

Qn 7(a)(i)

Year	1	2	3	4*	5
1	3	16	23	40	16
2	4	6	20	44	22
3	3	7	13	57	18
4	1	6	14	57	21
5	0	4	11	68	15
All	3	8	17	52	19

Qn 7(a)(ii)

Year	1	2*
1	38	50
2	31	60
3	30	63
4	31	65
5	25	72
All	32	61

Qn 7(a)(iii)

Year	3*	4*	5
1	38	33	20
2	45	28	18
3	53	27	15
4	53	31	13
5	60	25	11
All	49	29	16

Qn 7(a)(iv)

Year	1	2*
1	27	55
2	29	57
3	25	66
4	25	70
5	22	73
All	26	63

Qn 7(b) (% correct only)

Year	(i)	(ii)	(iii)	(iv)	(v)
1	84	73	18	65	23
2	84	76	25	69	27
3	89	80	34	78	37
4	89	85	38	83	43
5	91	91	44	84	51
All	87	80	30	75	35

Qn 8

Year	A	B	C*	D	E
1	4	4	11	8	73
2	4	4	11	6	74
3	1	4	16	6	73
4	2	4	12	5	77
5	0	3	21	6	69
All	3	4	14	7	73

Qn 9

Year	1	2	3	4	5*	6	Other	Omit
1	19	6	5	3	46	8	9	3
2	17	5	2	2	51	11	8	2
3	13	4	3	1	62	9	7	2
4	9	2	1	1	68	11	6	2
5	9	2	2	1	70	11	4	1
All	14	4	3	2	58	10	7	2

Qn 10

Year	Correct	Correct with Repeats	Incomplete	Omit
1	42	7	42	8
2	51	7	34	9
3	64	5	25	6
4	69	3	25	3
5	81	2	13	3
All	59	5	30	6

QN11

Year	A	B	C*	B and C	'equal'
1	11	12	26	23	27
2	13	12	26	20	29
3	13	14	24	25	24
4	13	16	16	21	33
5	12	15	18	21	34
All	12	14	23	22	29

QN12

Year	A	B*	C	A and B	'equal'
1	14	31	15	19	21
2	14	30	13	19	23
3	15	29	13	25	18
4	18	24	14	19	25
5	14	23	12	23	26
All	15	28	14	21	22

QN13

Year	Correct	Ambiguous	Certain	Other	Omit
1	29	14	22	31	3
2	40	7	26	25	3
3	40	6	25	25	2
4	50	8	19	20	3
5	48	11	21	17	2
All	41	9	23	24	3

QN14

Year	Correct	Ambiguous	Impossible	Other	Omi
1	37	10	28	21	4
2	43	6	36	10	4
3	42	5	41	10	3
4	46	6	38	7	3
5	44	8	39	7	2
All	42	7	36	11	3

QN15

Year	Correct	Ambiguous	Other	Omit
1	41	1	38	20
2	55	2	26	18
3	62	2	23	12
4	74	2	15	10
5	73	1	22	5
All	60	2	25	14

QN16 (% correct only)

Year	(A)	(B)	(C)	(D)	(E)	(F)
1	35	68	62	75	75	91
2	34	73	69	82	79	92
3	26	81	72	87	86	95
4	26	81	87	87	89	97
5	25	87	78	94	93	99
All	30	77	71	84	84	94

QN17

Year	A	B	C*	D
1	17	29	49	4
2	22	26	49	3
3	20	26	53	1
4	26	22	50	1
5	21	18	59	2
All	21	25	52	3

Reason	Year	Area Concept	Counting Concept	Contiguity Idea: Equal	Contiguity Idea: Close Better	Contiguity Idea: Spread Better	Ratio Concept	Probability Concept	Other	Omit
	1	1	35	1	21	14	0	0	17	10
	2	3	34	2	21	18	0	1	16	7
	3	3	35	3	21	17	1	2	12	6
	4	4	31	3	19	22	1	1	8	11
	5	5	38	3	14	19	1	4	7	8
	All	3	35	2	20	18	1	1	12	8

Qn 18

Year	A	B*	C	D	E
1	7	40	12	38	3
2	6	45	9	38	3
3	3	61	5	30	0
4	4	65	3	25	2
5	2	70	3	22	2
All	5	55	7	31	2

Qn 19

Year	A	B*	C	D
1	32	43	17	7
2	27	49	18	6
3	19	61	16	4
4	18	63	16	3
5	14	69	14	3
All	23	56	16	5

Qn 19 Reason

Year	Area Concept	Counting Concept	'Equal'	Ratio Concept	Contiguity	Position or speed	Other	Omit
1	41	27	7	1	7	1	10	6
2	51	24	5	0	3	2	8	5
3	62	15	4	1	5	2	6	6
4	67	14	4	0	4	1	5	4
5	69	13	2	2	4	0	3	6
All	57	19	4	1	5	1	7	5

Qn 20(a)

Year	Clare*	Susan	'Cannot tell'	Omit
1	38	53	1	7
2	34	56	2	7
3	30	59	4	7
4	40	46	3	9
5	35	49	5	9
All	35	53	3	8

Qn 20(b)

Year	% Correct (cols 1+7)	1 2 Pattern Regularity C	S	3 4 Inequality of 0,1 C	S	5 6 Equality of 0,1 C	S	7 8 Run Lengths C	S	Other	Omit
1	21	20	2	8	21	2	1	1	15	17	13
2	21	17	3	7	21	1	0	4	18	13	14
3	23	21	4	4	17	3	0	2	21	14	14
4	28	26	3	3	14	4	0	2	20	16	11
5	28	24	6	1	16	4	0	4	19	12	12
All	24	21	3	5	18	3	1	3	18	14	13

Qn 21(a)

Year	A*	B	C	D	E
1	48	21	12	8	11
2	52	22	11	7	7
3	57	18	10	6	8
4	57	20	8	3	10
5	65	14	5	3	11
All	55	19	10	6	9

Qn 21(b)

Year	A*	B	C	D	E
1	34	17	23	17	8
2	37	16	17	22	8
3	44	13	13	20	9
4	46	11	12	17	12
5	53	11	11	13	12
All	42	14	16	18	10

Qn 21(c)

Year	A	B	C*	D	E
1	13	3	61	18	4
2	12	3	69	11	3
3	12	1	68	12	5
4	10	2	67	14	7
5	12	0	71	10	6
All	12	2	67	13	5

Qn 21(d)

Year	A	B*	C	D	E
1	29	36	10	18	6
2	24	37	11	21	5
3	22	41	7	24	5
4	18	42	5	29	5
5	17	50	5	22	5
All	23	40	8	22	5

Qn 22

Year	'1 and 2'*	'1 or 2'	5/6/7/8	'Same'	Other	Omit
1	5	7	22	57	6	3
2	4	5	20	62	6	2
3	7	3	19	62	7	3
4	9	7	16	59	7	3
5	13	4	15	59	5	3
All	7	5	19	60	6	3

Qn 23

Year	A	B*	C	D	E
1	9	15	8	9	58
2	8	16	9	6	60
3	5	15	8	6	65
4	4	18	8	4	65
5	3	20	7	4	66
All	6	17	8	6	62

Qn 24

Year	A	B*	C	D
1	18	7	8	66
2	16	9	5	69
3	25	7	4	63
4	33	8	3	54
5	41	6	3	49
All	25	8	5	61

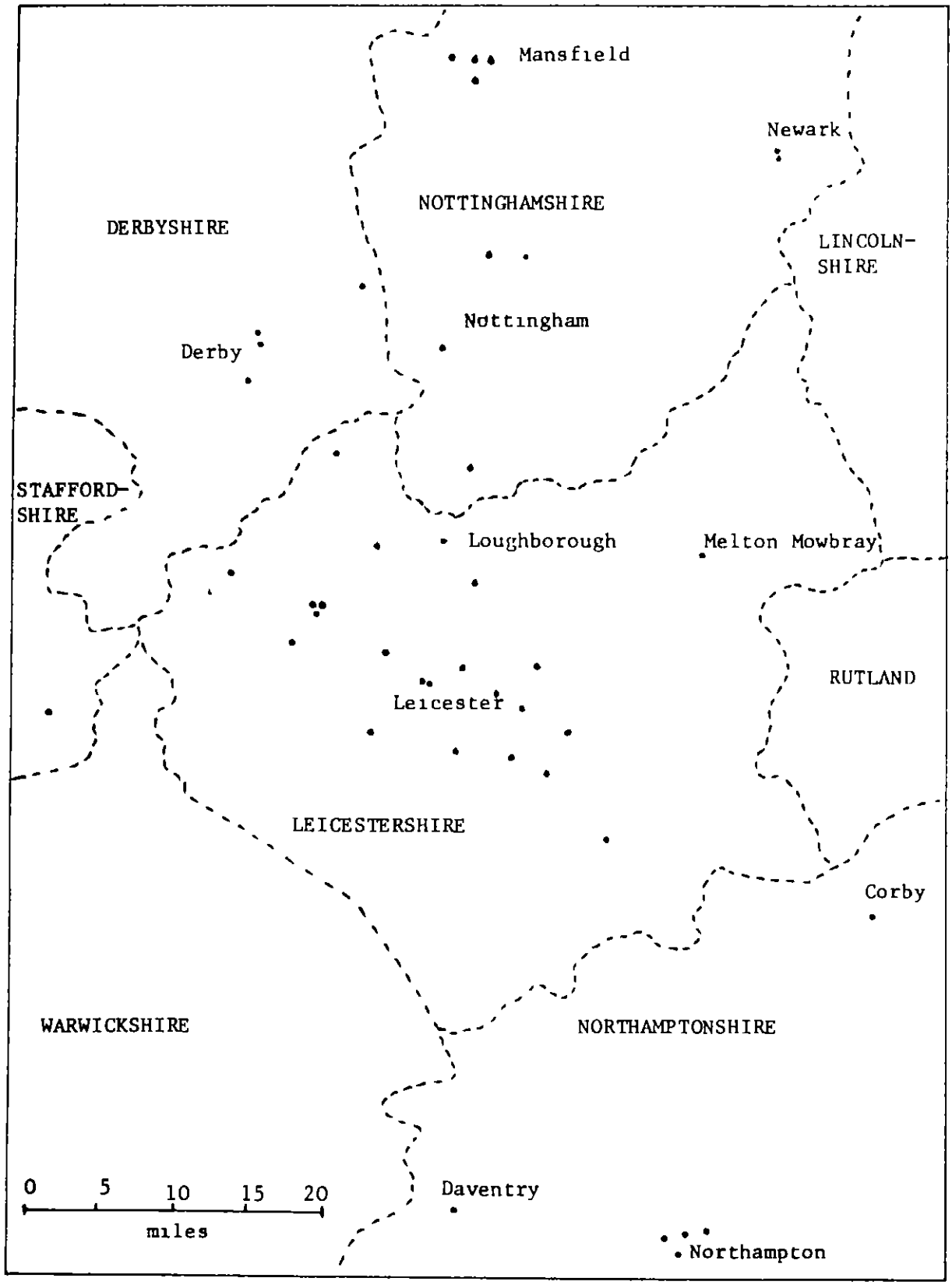
Qn 25

Year	A*	B	C
1	13	63	24
2	13	68	17
3	13	73	13
4	18	68	12
5	25	63	10
All	16	67	16

Qn 26 (% correct and omits only)

	(a)/(b)		(c)		(d)	
Year	6	Omit	24	Omit	120	Omit
1	64	4	5	17	1	21
2	68	3	9	16	2	21
3	81	1	15	8	2	13
4	80	2	19	9	9	15
5	87	4	25	9	9	13
All	75	3	14	12	4	17

APPENDIX C



Locations of the 44 schools in the Main Sample

NOTE on GCE O Level and CSE Examinations

The GCE O Level Examination is designed for the top 20% of pupils in English schools, and is normally sat at 16 years of age.

The CSE Examination was intended for the 40% of pupils immediately below the top 20%. However it is very often sat by pupils of much lower ability and sometimes by pupils of higher ability. The age at which it is sat is 16 years (i.e. after 5 years secondary education).

For both CSE and GCE O Level the probability questions set are of a formal computational nature, involving sum and product calculations and the use of tree diagrams.

Various odd documents. 9th

CHANCE & PROBABILITY CONCEPTS PROJECT

Details of Classes we wish to Test

Age	Secondary School Level	Years 1 to 5 inclusive (age 11 - 16 years)
Ability	We are interested in testing right across the ability range. Wherever possible we would like to test mixed ability groups.	
Probability	We do not mind whether or not pupils have been formally taught probability. Our test makes no reference to the term 'probability'.	
Mathematics	We do not mind if the pupils are in mathematics sets, tutor groups, streams, general studies groups, etc.	
Teacher's Assessment	It would be very informative for us if the school could provide a broad assessment of each pupil's mathematical ability. How this can be best done will be discussed when we visit.	
Teaching Style	We are interested to see if there is any correlation between teaching style experienced (including ~~tests~~ books used) and understanding of probability concepts. We hope to obtain that information during our visits.	

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CHANCE & PROBABILITY CONCEPTS PROJECT

Teaching Style Questionnaire

1. School: _____ Date: _____

Class: _____

Set/Stream (in relation to whole year group) _____

2. Textbook(s) used: _____

When was work on Probability last done: _____

What topics were covered? _____

When was work on Fractions last done: _____

What topics were covered? _____

3. What average % of lesson times is devoted to:

(a) Presentation of new material, including worked examples _____

(b) Pupils solving problems working in groups _____

(c) Pupils solving problems individually _____

(d) Going over homework, going over difficulties, etc. etc. _____

4. What average % of lesson times is devoted to 'practical' work using apparatus, or activities (e.g. using compasses, rulers, counters, scales, dice, geoboards, taking censuses) _____

1) Details of the Tests

Probability Concepts Test — Duration: 60 minutes

This comprises 26 questions which seek to establish the level of understanding of the pupil on random events, permutations, combinations and arrangements. No knowledge of formal 'probability' is assumed.

Each pupil will be given the 16 page Test Booklet. For all pupils in years 1,2,3 (i.e. aged 11+, 12+, 13+) the test will be read out. This is to minimise reading difficulties, to ensure appropriate emphasis of the important ideas in the questions, and to ensure that each question receives the same attention from each pupil. Our experience with pilot versions of the test indicate that 60 minutes is adequate for all groups. We shall not normally read out the test questions to pupils in years 4,5 (i.e. aged 14+, 15+) unless the school recommends doing so.

Intelligence Test — Duration: Introduction 18 minutes, Test 42 minutes

The chosen test is called AH2. It is published by the National Foundation for Educational Research (NFER) and is based on the work of Dr A W Heim and others at the Cambridge Psychology Laboratory. The test is in three parts:

Part 1	Verbal	(15 minutes)
Part 2	Numerical	(15 minutes)
Part 3	Perceptual	(12 minutes)

Norms are available for secondary school pupils for each separate section and overall.

We are using this test for two main reasons:

a) to ensure the 'normality' of our sample
b) to investigate the relationship between probability conceptual level and Verbal, Numerical and Perceptual ability.

Our original intention was to use a similar test (AH4) but AH2 has just been made available to us and is better for our purposes, outweighing the inconvenience of its being more lengthy to administer than AH4.

2) Results of the Tests

If the school wishes we will forward to the school the test results of individual pupils, although we recommend that they not be divulged to the pupils themselves.

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SCHOOL: _____ CLASS: _____

	NAME	MATHEMATICAL ABILITY	GRADE	PROB TEST	N	V	P	AB2
1		1 2 3 4 5 6 7 8 9 10						
2		1 2 3 4 5 6 7 8 9 10						
3		1 2 3 4 5 6 7 8 9 10						
4		1 2 3 4 5 6 7 8 9 10						
5		1 2 3 4 5 6 7 8 9 10						
6		1 2 3 4 5 6 7 8 9 10						
7		1 2 3 4 5 6 7 8 9 10						
8		1 2 3 4 5 6 7 8 9 10						
9		1 2 3 4 5 6 7 8 9 10						
10		1 2 3 4 5 6 7 8 9 10						
11		1 2 3 4 5 6 7 8 9 10						
12		1 2 3 4 5 6 7 8 9 10						
13		1 2 3 4 5 6 7 8 9 10						
14		1 2 3 4 5 6 7 8 9 10						
15		1 2 3 4 5 6 7 8 9 10						
16		1 2 3 4 5 6 7 8 9 10						
17		1 2 3 4 5 6 7 8 9 10						
18		1 2 3 4 5 6 7 8 9 10						
19		1 2 3 4 5 6 7 8 9 10						
20		1 2 3 4 5 6 7 8 9 10						
21		1 2 3 4 5 6 7 8 9 10						
22		1 2 3 4 5 6 7 8 9 10						
23		1 2 3 4 5 6 7 8 9 10						
24		1 2 3 4 5 6 7 8 9 10						
25		1 2 3 4 5 6 7 8 9 10						
26		1 2 3 4 5 6 7 8 9 10						
27		1 2 3 4 5 6 7 8 9 10						
28		1 2 3 4 5 6 7 8 9 10						
29		1 2 3 4 5 6 7 8 9 10						
30		1 2 3 4 5 6 7 8 9 10						
31		1 2 3 4 5 6 7 8 9 10						
32		1 2 3 4 5 6 7 8 9 10						
33		1 2 3 4 5 6 7 8 9 10						
34		1 2 3 4 5 6 7 8 9 10						
35		1 2 3 4 5 6 7 8 9 10						

CARD DETAILS

Each record — the data for one subject — was recorded on two cards — CARD A, CARD B

For one subject the first 8 character positions on CARD A and CARD B would be the same, providing a unique identifier. The cards were all held in correct order CARD A followed by CARD B, for each subject

Details of data stored on each card are given on the following 3 sheets.

The cards corresponded to five school years:

YEAR 1	640 records	(1280 cards)
YEAR 2	670 records	(1340 cards)
YEAR 3	670 records	(1340 cards)
YEAR 4	540 records	(1080 cards)
YEAR 5	410 records	(820 cards)

CARD A

COL.			
1-2	School Code	NN	10 to 71
4	Year Code	N	1 to 5
6-8	Pupil Code	NNN	1 to 150 (?)
11	Qn 1		
12	Qn 2		
14	Qn 3 answer		
15	Qn 3 reason		
16	Qn 4		
17	Qn 5		
19	Qn 6(a) answer		
20-21	Qn 6(a) reason		
22	Qn 6(b) answer		
23-24	Qn 6(b) reason		
26	Qn 6(c) answer		
27-28	Qn 6(c) reason		
29	Qn 6(d) answer		
30-31	Qn 6(d) reason		
32	Qn 6(e) answer		
33-34	Qn 6(e) reason		
36	Qn 7(a)(i)		
37	Qn 7(a)(ii)		
38	Qn 7(a)(iii)		
39	Qn 7(a)(iv)		
41	Qn 7(b)(i)		
42	Qn 7(b)(ii)		
43	Qn 7(b)(iii)		
44	Qn 7(b)(iv)		
45	Qn 7(b)(v)		
47	Qn 8		
48	Qn 9		
49	Qn 10		
50	Qn 11		

CD3

CARD A

COL.	
53	Qn 13
54	Qn 13
55	Qn 14
56	Qn 14
57	Qn 15
58	Qn 15
59	Qn 15
60	Qn 16(A)
61	Qn 16(B)
62	Qn 16(C)
63	Qn 16(D)
64	Qn 16(E)
65	Qn 16(F)
66	Qn 17 answer
67	Qn 17 reason
68	Qn 18
69	Qn 19(a)
70	Qn 19(b)
71	Qn 20(a)
72	Qn 20(b)
73	Qn 21(a)
74	Qn 21(b)
75	Qn 21(c)
76	Qn 21(d)
77	Qn 22
78	Qn 23
79	Qn 24
80	Qn 25

CARD B

COL			
1-2	School Code	NN	1 to
4	Year Code	N	1 to 5
6-8	Pupil Code	NNN	1 to
10-15	Probability Test Date	ddmmyy	010980 to 310381
17	Sex	N	1 for M, 2 for F
19-20	Age (years) on taking Prob. Test	NN	11 to 16
22-27	Pupil's date of birth	ddmmyy	
29-30	Maths. Set	NN	
32-33	Maths. Ability	NN	1 to 10
36	Qn 26(a)	N	
38-39	Qn 26(b)	NN	
41-42	Qn 26(c)	NN	
44-46	Qn 26(d)	NNN	
51	Short or long AH2 test administered	[can be ignored]	
53-54	AH2 Verbal Score	NN	0 to 40
56-57	AH2 Numerical Score	NN	0 to 40
59-60	AH2 Perceptual Score	NN	0 to 40
62-64	AH2 Total Score	NNN	0 to 120
71-76	AH2 test date	ddmmyy	
78-79	Age (years) on taking AH2 test	NN	11 to 16

MARKING SCHEME

Sept 1982

Qn.	Ans	Mark	Subtest
1.	C	1	PB
2.	B	1	PB
3.	B	1	PB
3. reason		1	PB
4.	same	1	PB
5.	C	1	PB
6 (a)	A	1	PB
6 (a) reason		-	
6 (b)	C	1	PB
6 (b) reason		-	
6 (c)	C	1	PB
6 (c) reason		-	
6 (d)	B	1	PB
6 (d) reason		1	PB
6 (e)	A	1	PB
6 (e) reason		1	PB
7 (a) (i)	4	1	VB
	or 3	½	VB
(ii)	2	1	VB
(iii)	3	1	VB
(iv)	2	1	VB
(b) (i)		1	VB
(ii)		1	VB
(iii)		1	VB
(iv)		1	VB
(v)		1	VB
8.	C	1	PB
9.	5p	1	CB

Qn	Ans.	Mark	Sub
10.	H T T H T T H H	1 ignore repeat	CC
11.	C	1	PB
	or B and C	½	
12	B	1	PB
	or A and B	½	
13		1	VB
14		1	VB
15		1	VB
16	B, E	3 reduced by 1 each for any extra ticke or if either above missed	
17.	C	1	PB
17	reason	-	
18	B	1	PB
19 (a)	B	1	PB
19 (b)		1	PB
20 (a)	Clare	1	PB
(b)		1	PB
21 (a)	A	1	PB
21 (b)	A	1	PB
21 (c)	C	1	PB
21 (d)	B	1	PB
22	1 and 2	1	PB
	or 1 or 2	½	
23	B	1	PB

MARKING SCHEME
Sept 1982

Qn	Ans.	Mark	Subtest
24	B	1	PB
25	A	1	PB
26 (a)	ABC, ACB, BAC, BCA, CAB, CBA	1 (ignore repeats)	CB
(b)	6	-	
(c)	24	1	CB
(d)	120	1	CB

VB	total	15	(Verbal subtest)
CB	total	5	(Combinatoric subtest)
PB	total	30	(Probabilistic subtest)
		50	

CODES WHICH CORRESPOND TO CORRECT ANSWERS

QN.	CODE	
1	3	
2	2	
3	2	3 reason 1,4,5
4	7	
5	3	
6a	1	see separate sheet for 'reasons'.
6b	1	
6c	3	
6d	2	
6e	1	
7a i	4	
7a ii	2	
7a iii	3,4	
7a iv	2	
7b i	1,2,3,4	
7b ii	1,3,5	
7b iii	1	
7b iv	1,6	
7b v	1,3	
8	3	
9	5	
10	1,3	
11	3	
12	2	
13	1,4	
14	1	
15	1,2	
16a	0	
16b	1	
16c	0	
16d	0	
16e	1	
16f	0	
17	3	
17 reason	1,2,3,4,5	
18	2	
19	2	
19 reason	1,2,4	
20a	1	
20b	1,7	
21a	1	
21b	1	
21c	3	
21d	2	
22	1,2	
23	2	
24	2	
25	1	
26a	1	
26b	6	
26c	24	
26d	120	

CODES WHICH CORRESPOND TO CORRECT ANSWERS

FOR REASONS FOR QN 6

6a 1 to 6, 8, 19, 26, 28

6b 1 to 6, 8, 16, 26, 28

6c 1, 2, 7, 8, 10, 22, 24, 27, 28, 29, 31

6d 1, 2, 15 to 18, 26, 30 to 33

3 reason

Valid answers would discuss: area, size, space, angle or number of sectors, or use fraction, ratio, probability

Invalid answers would discuss: position of pointer, or speed of turn, or say they are the same since both have a 3, or invoke luck, etc., or merely say "because it is better" etc.

6 (d) reason

Valid answers would discuss: ratio or proportion, perhaps vaguely or with actual numbers,
E g.
"G because it has 3 times as many"
"G because it has less than half White"
"G because it has more Black compared to White"
"The odds are greater for G"

Invalid answers would discuss: difference in numbers of Black only, or Black – White differences, luck, etc

NOTE The interpretation of the reason usually needs reference to the actual box ticked – a good guide to what the subject is thinking.

6(e) reason

Valid answers would discuss : ratio or proportion in some form, perhaps vaguely,
Eg.
" They each have 3 times as many Black"
" They both have the same Black to White"
" K is double J"

Invalid answers would discuss : difference in numbers of Black only, or Black – White differences, or simply say "they are the same" or "both have Black and White", or refer to luck, etc.

NOTE The interpretation is aided by referral to the actual box ticked.

7(b)

(i) straightforward to mark

(ii) Valid : can happen etc
fairly likely, might etc

Invalid : certain, very likely etc
very unlikely, impossible etc

(iii) to be valid this must specify a 50% likelihood in some way. Thus " might or might not" etc will not do, but "as likely as not" will do.

7 (b) (iv) Valid: unlikely, hard, rare, might, could et
Invalid: no chance, impossible, likely etc

(v) Valid: easy, very likely etc
Invalid: sure, does, will etc, likely, possible unlikely, impossible, certain etc.

13, 14 It is not possible to lay down hard and fast rule here. Often answers are impossible to judge since the context is unknown The benefit of the doubt must be given in such cases. Many, however, put certain and impossible events for very likely and unlikely, which are to be marked wr

15 Almost everything in this world can be considered to have some chance element in it. Mark correct only those which are predominantly chance events. Do not give the benefit of the doubt for this question

20 (b) Valid reasons will discuss: Clare too regular or too short runs E.g. "they seem to be in a sequence" or "she never gets more than three Os together".

MARKING CODES

Qu

1.	0	OMIT
	1	A
	2	B
	3	C
	4	D
	5	E
	8	other (e.g more than one ticked)
	9	illegible

2. as for 1

3. answer as for 1

3 reason

0	OMIT
1	area / size of spaces / angle (area concept)
2	numbers of spaces (counting concept)
3	no difference / same chance (equality)
4	fraction / ratio idea comparing discs
5	probability specified
6	position of spinner initially significant, or speed (e.g. "harder that side", "depends how hard / fast", "its near")
7	saying nothing (e.g. "because its better" / "can't tell")
8	other (e.g. "both have a 3" / "its a smaller disc")
9	illegible / unintelligible.

4

0	OMIT
1	1
2	2
3	3
4	4
5	5
6	6
7	"All the same"
8	Combination which includes 6
9	Other

5 as for 1

6(a) answer as for 1

6(a) reason

0	omit
1	ratio / fraction idea
2	probability
3	more Black in A
4	more Black in A and same White in Both
5	less Black in B
6	less Black in B and same White in both
7	same White in both
8	difference between colours in A(2) > diff in B(1)
9	illegible
10	less counters in B / less to choose from

11 more blacks than whites
12 luck / can't say / can't see in bag
13 more counters (in A) / more balls to choose from
14 other
15 both have black counters to choose from / each has a white
16 saying nothing
17 nonsense
18 B > A in both
19 more B to W
20 positional argument
21 saying nothing new (involving ratio or probability)
22 it has only 1 white

6(b) answer as for 1

6(b) reason

0 omit
1 ratio / fraction idea
2 probability
3 more White in D
4 more White in D and same Black in both
5 less White in C / only 2 white
6 less White in C and same Black in both
7 same Black in both
8 difference between colours in C (3) > difference in D (2)
9 illegible
10 more Black counters (in C / in D)
11 more Black than White in both

12 saying nothing
13 same number of balls / just as many in both / both the same
14 other (e.g. 'no chance')
15 less/more Black compared to White / more B and less W
16 less chance for White
17 more counters (in D)
18 position
19 both have White Black / both have more B / more B than (a)
20 nonsense
21 less White than Black
22 luck / can't tell / can't look
23 less balls in C
24 more B than W
25 ratio idea, but no comparison made

6(c) answer as for 1

6(c) reason

0 omit
1 ratio / fraction idea
2 probability
3 more Black in F
4 less Black in E
5 more White in F
6 less White in E
7 difference between bags same (2 more B in F, 2 more W in F)
8 difference within bags same (B − W = 0 in both bags)
i.e. no difference within bags
9 illegible
10 same amount in each / equal B and W / bags equal / even / same odds.

11	it has same no of B and W
12	equal/same / better / worse chance
13	position
14	nonsense
15	both have Black
16	less counters
17	more counters
18	saying nothing
19	can't tell / Black
20	more Black than White
21	more Black but more counters
22	$\frac{1}{2}$ chance
23	other
24	F is double E
25	saying nothing new (ratio or probability)

6(d) answer ... as for 1

6(d) reason

0	Omit
1	fraction / ratio idea
2	probability
3	more B in H
4	less B in G
5	more W in H
6	less W in G
7	difference between bags [8 > 6]
8	difference within bags [8 < 10]
9	–
10	'MORE' or 'LESS'
11	it has B > W
12	only 4W in G / a few W / lots less W
13	B > W in both

14 OTHER
15 less than half are W (in G)
16 more B compared to W / less W compared to B
17 less chance for W
18 less W and more B / less B and more W
19 more B in G / less W in H
20 BOTH have B / BOTH have W / BOTH have B > W
21 saying nothing new
22 nonsense
23 3 times as many / $\frac{1}{3}$ as many
24 half W as B / twice B as W
25 LESS counters
26 odds greater for G
27 same / more or less same
28 LUCK
29 saying nothing new (ratio / probability)
30 more B per WC
31 higher ratio in G (not specified)
32 lots B / much more B than W
33 difference greater
34 higher ratio in H

6(e) answer as for 1

6(e) reason

0 Omit
1 fraction / ratio
2 probability
3 more B in K
4 less B in J
5 more W in K
6 less W in J

7 difference between bags [3 > 1]
8 difference within bags [2 < 4]
9 —
10 K MORE / J LESS
11 it has B > W
12 same W
13 less W
14 OTHER
15 equal nos. / same
16 same / better / worse ---- chance (or probability)
17 more counters
18 less W and more B (etc)
19 more B compared to W (etc)
20 BOTH have B / BOTH have W / BOTH have B > W
21 only 1 W
22 saying nothing new
23 3 times / $\frac{1}{3}$ as many
24 nonsense
25 not many W
26 only 2 W
27 both have same B and W / same B compared to W
28 K is double J / same but doubled
29 LUCK
30 more B
31 saying nothing new (ratio)
32 equal ratio / probability (not specified)
33 bigger proportion
34 difference greater
35 lots more B (than W)

7(a) (each part)

0	OMIT
1 to 5	1 to 5
8	other (e g two ticks)
9	illegible

7(b) (i)

0	omit / saying nothing
1	cannot happen / cannot be done / cannot / will not / could not
2	not possible, unable
3	no chance / not a chance / no way
4	never (happens)
5	possible / can / could happen / likely
6	unlikely / hardly likely / not likely / not very likely / hard
7	not often / not very often
8	hard / very hard
9	illegible or other wrong

7(b) (ii)

0	omit / saying nothing
1	can happen / can be done / can / could / may / might / feasible / sometimes / maybe
2	unlikely / not often
3	likely / often / quite often / fair chance / good chance / every chance / probable / easy
4	more than likely / very likely / very often / nearly every time / should happen / almost certain / easy / probable
5	other correct answers
6	always happens / certain / every time, happens
7	never happens / will not / impossible
8	50-50 chance, or other specified level
9	illegible or other wrong

7(b) (iii)

0 – omit

1 probability = $\frac{1}{2}$ /50-50/ half and half / 50% / evens / half the time / as likely as not

2 equal / same probability / same / odds same / fair / draw / either / average

3 impossible

4 unlikely (1 in 10)

5 possible / might / might or might not / sometimes / can't say

6 likely / very likely / nearly every time / usually / quite often / good chance

7 certain / always / sure

8 other wrong

9 illegible / saying nothing

7(b) (iv)

0 omit

1 unlikely / hard / not often / rare / doubtful / don't think / not very possible / just might / slim chance

2 not a hope / no chance / would not happen / not really / impossible

3 might / might or might not / could / sometimes / may be / depends

4 quite often / likely / nearly every time / big chance

5 certain

6 specific probability or ratio $< \frac{1}{2}$

7 specific probability or ratio $\geq \frac{1}{2}$

8 other wrong

9 illegible / saying nothing

7(b) (v)

0 omit / saying nothing
1 very likely / good chance / more than likely / easy / should
2 always happens / certain / sure / bound to happen / does / yes / no doubt / will
3 likely / quite likely / very possible / quite easy
4 sometimes / possible / might / can be done / no one knows
5 unlikely / doubtful / just might / don't think so
6 impossible
7 specific probability $> \frac{1}{2}$
8 specific probability $\leq \frac{1}{2}$
9 illegible / other wrong.

8

0 OMIT
1 to 5 A to E
8 other
9 illegible

9

0 OMIT
1 to 6 1 to 6
8 Other
9 illegible / unintelligible

10	0	OMIT
	1	Correct (Exactly 4 different pairs — allow HT repeated once)
	2	Correct but repeats
	3	Correct but with 'extras' (e.g. "on edge")
	6	Other
	7	Repeats but not all 4 occurring
	8	Too few (usually just HT and TH)
	9	illegible / unintelligible

11	0	OMIT
	1	A
	2	B
	3	C
	4	B and C
	5	each -- as likely
	8	other
	9	illegible

12 as for 11 except 4 --- A and B

13

FIRST CODE DIGIT

Correctness

0 OMIT
1 correct (ie very likely)
2 undecidable / matter of conjecture
3 certain / sure / true
4 moderately likely
5 unlikely
6 impossible
7 not true
8 other (hypothetical sentence with obviously no attempt to make it sensible or valid)
9 illegible / nonsense

SECOND CODE DIGIT

Content

0 - OMIT
1 - wealth
2 - abdication Succession
3 - servants/fu
4 - longevity/dea ill
5 - ceremony/rega
6 - visiting/trav
7 - normal life for Queen
8 - other
9 - nonsense

14

FIRST CODE DIGIT

Correctness

0 - OMIT
1 - correct (i e unlikely)
2 - undecidable / matter of conjecture
3 - certain / sure / true
4 - likely
5 - so unlikely as to border on impossible
6 - impossible
7 - not true / will not happen
8 - other (hypothetical sentence)
9 - illegible / nonsense

SECOND CODE DIGIT

(as for 13)

15

FIRST CODE DIGIT

Correctness of answer

0 OMIT
1 quite correct (include weather here)
2 basically correct — mainly involving chance element (e.g fires, most accidents, pools, gambling)
3 not really correct — skill or carelessness important elements (eg cutting yourself out
4 wrong — even quite predictable or controllable
6 undecidable (not clear what aspect is implied)
8 other (hypothetical sentence)
9 illegible / nonsense

SECOND CODE DIGIT

Content of answer

0 OMIT
1 Dice / Cards / Coins / Raffle / Bingo / Premium Bond / Roulette / Boy or girl
2 Accidents / Illness / Death
3 Finding / Losing
4 Pools and other gambling / Games / Winning Ratio
5 Meeting people / Coincidences / Luck
6 Weather / Natural events
7 Breakdowns / explosions / Mechanical
8 Other / Irrelevant (if 8 for first code)
9 illegible / nonsense

15 THIRD CODE DIGIT

Rarity of event

0	OMIT	
1	common	(e.g. rain, dice landing 3) ($p \geq 1/6$)
2	medium	(snow, falling over, accidents) ($p < 1/6$)
3	rare	(car crash, winning raffle) ($p < 1/20$)
4	very rare	(pools win)
5	impossible	
6	undecidable	
8	irrelevant	
9	illegible / nonsense	

16 (all parts)

0 box not ticked
1 box ticked

17

0	OMIT
1	area / size of spaces / angle (area concept)
2	number of spaces / sides (counting concept)
3	contiguity discussed leading to "same" (balance
4	fraction / ratio idea comparing spinners
5	probability specified / "50-50 chance"
6	2s close better - contiguity concept.
7	2s spread better - non-contiguous
8	"no difference" / saying nothing (i.e no reason given
9	illegible / other wrong

18 as for 8

19(a) as for 1

19(b)

0	OMIT
1	area / size of spaces / angle (area concept)
2	number of spaces / sides (counting concept)
3	"no difference" / "same chance" / saying nothing
4	fraction / ratio idea comparing discs
5	probability specified / "half chance each"
6	"luck" / "can't say" / "depends"
7	contiguity concept (close / spread)
8	position of pointer initially a factor / speed
9	illegible / other wrong

20 (a)

0	OMIT
1	Clare
2	Susan
3	both
4	can't say / no one can tell
8	other
9	illegible / unintelligible

(b)

0		OMIT
1	PATTERN	Clare too regular pattern (or Susan!)
2	PATTERN	Susan too irregular (or Clare!)
3	UNEVEN	Too many 1s/0s in Clare's / Too far from 50%
4	UNEVEN	Too many 1s/0s in Susan's / Too far from 50%
5	EVEN	Clare too close to 50%, of each
6	EVEN	Susan too close to 50% of each
7	RUNS	Clare too short runs
8	RUNS	Susan too long runs
9		Illegible / other wrong / saying nothing (ie no reason) both cheating

21 (each part) as for 8

22	0	OMIT
	1	1 and 2 (include here "1 or 2")
	2	1, or 2 but not both, and no others
	3	1, 2, 3 and 4 specified together
	4	any others involving <u>both</u> 1 and 2
	5	3, 4, or either
	6	5, 6, 7, 8, or any subset
	7	"all the same"
	8	other (e.g. "1 and 8")
	9	illegible/unintelligible

23 as for 8

24 as for 1

25	0	OMIT
	1	A
	2	B
	3	C
	4	"depends on initial contents of bags"
	8	other
	9	illegible

26 (a) 0 OMIT
1 correct (i.e precisely six distinct permutation with no repeats — except possibly ABC once)
2 correct but repeats occurring
8 other (wrong)
9 illegible

(b) 0 OMIT , otherwise record number with 99 maxim
(c) 0 OMIT , otherwise record number with 99 maxim
(d) 0 OMIT , otherwise record number with 999 maxim

N.B. If "a lot" put maximum. If too large put maximum.
If more than one, or a range specified, take best or average.